

Name _____

CALCULUS SUMMER PACKET

Please be sure to show work when necessary. Feel free to use the internet or any other resource to help you. You will be assessed on the material at the beginning of the school year after a quick review.

1. Given $f(x) = x^2 - 2x + 1$, find $\frac{f(x)-f(1)}{x-1}$.

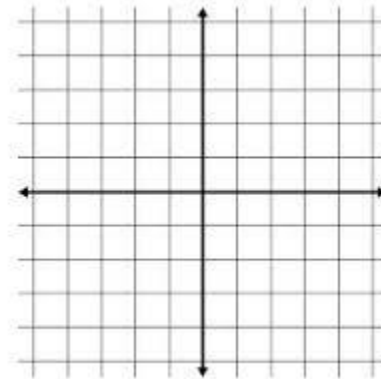
2. Given $f(x) = \frac{1}{x}$, find $\frac{f(x+\Delta x)-f(x)}{\Delta x}$.

3. Sketch a graph of the function and find its domain and range.

$$f(x) = \begin{cases} 5 - x^2 & x < 2 \\ |x - 4| & x \geq 2 \end{cases}$$

D= _____

R= _____



4. Find:

a. $\lim_{x \rightarrow \infty} \frac{1}{x}$

b. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$

c. $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$

d. $\lim_{\theta \rightarrow 0} \frac{\cos \theta \tan \theta}{\theta}$

e. $\lim_{x \rightarrow 3^+} \frac{|x-3|}{x-3}$

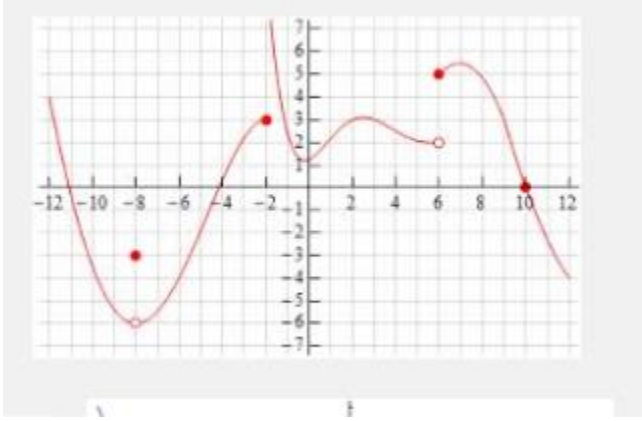
5. Use the graph to determine:

a) $\lim_{x \rightarrow -8} f(x)$

b) $\lim_{x \rightarrow 6} f(x)$

c) $\lim_{x \rightarrow -2^-} f(x)$

d) $f(-8)$



c) Discuss the continuity of the function. (give reasons for each point of discontinuity)

6. Find the constant a such that the function is continuous on the entire real line.

$$g(x) = \begin{cases} x^2 - a^2 & x \neq a \\ 8 & x = a \end{cases}$$

7. True or false: Give a brief explanation.

a. The graph of a rational function has at least one vertical asymptote.

b. If $\lim_{x \rightarrow c} f(x) = L$, then $f(c) = L$.

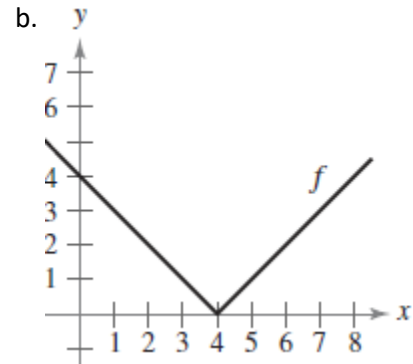
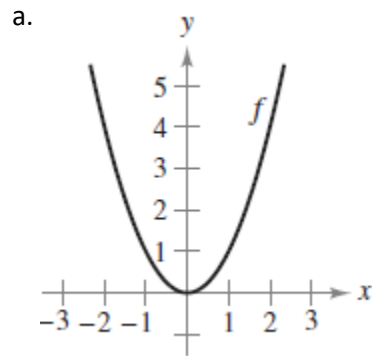
c. If a function is continuous at a point, then it is differentiable at that point.

10. Using the definition of derivative $\left(f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \right)$, find the derivative of:

a. $f(x) = 4x^2$

c. $f(x) = \frac{1}{x-2}$

11. The graph of f is given. Sketch the graph of the derivative on the same graph.



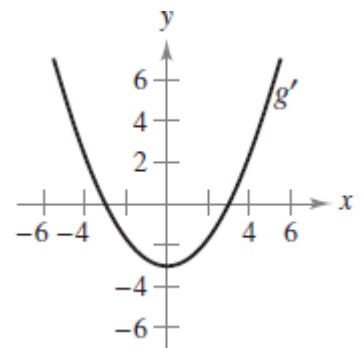
12. Identify a function f that has $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$. Sketch the function.
- $f(0) = 4; f'(0) = 0;$

13. The figure shows the graph of g' :

a. $g'(0) =$

b. $g'(3) =$

c. What can you conclude about the graph of g knowing that $g'(1) = \frac{-8}{3}$?



d. Is $g(6) - g(4)$ positive or negative? Explain.

e. Is it possible to find $g(2)$ from the graph? Explain.

14. Find the derivative of the function (using the rules).

a. $y = 3x^2 - 5\sqrt[3]{x} + 6 + \frac{4}{x}$

b. $f(x) = \frac{2x + x^3 - x^5}{4x}$

c. $f(x) = \sin^2 x + \cos^2 x$

d. $y = 5 \sin x - 6 \tan x$

e. $f(x) = 3x \left(4x^2 - 5x + \frac{1}{x^2} \right)$

15. Find the equation of the line that is tangent to $f(x) = 6x^3 - 2x$ at $(1, 4)$. Use derivative rules.

16. Determine the point(s) at which the graph of $f(x) = x + \sin x$ $0 \leq x < 2\pi$ has a horizontal tangent line.

17. Approximate the area under $f(x) = x^2 + 1$ on $[-4, 0]$ using 4 rectangles whose height is given using the right endpoint.

18. Let f be the function defined below. Which of the following statements about f are true?

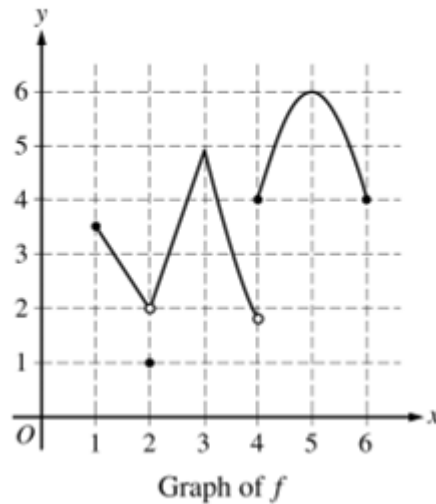
$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

- I. f has a limit at $x=2$
- II. f is continuous at $x=2$
- III. f is differentiable at $x=2$

- A) I only B) II only C) III only D) I and II only E) I, II, and III

19. The graph of the function f is shown below. Which of the following statements is false?

- A) $\lim_{x \rightarrow 2} f(x)$ exists B) $\lim_{x \rightarrow 3} f(x)$ exists C) $\lim_{x \rightarrow 4} f(x)$ exists D) $\lim_{x \rightarrow 5} f(x)$ exists



20. $\lim_{x \rightarrow \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

- A) -3 B) -2 C) 2 D) 3 E) nonexistent