Name_____

BC CALCULUS SUMMER PACKET

Please be sure to show work when necessary. Feel free to use the internet or any other resource to help you. You are expected to know this material and will be assessed on it at the beginning of the school year after a quick review.

1. Given
$$f(x) = x^2 - 2x + 1$$
, find $\frac{f(x) - f(1)}{x - 1}$.

2. Given
$$f(x) = \frac{1}{x}$$
, find $\frac{f(x+\Delta x)-f(x)}{\Delta x}$.

3. Sketch a graph of the function and find its domain and range.



4. Find:

a.
$$\lim_{x \to \infty} \frac{1}{x}$$
 d. $\lim_{\theta \to 0} \frac{\cos\theta \tan\theta}{\theta}$

b.
$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$
 (hard one) e. $\lim_{x \to 3^+} \frac{|x-3|}{x-3}$

c.
$$\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$$

5. Find the constant *a* such that the function is continuous on the entire real line.

$$g(x) = \begin{cases} \frac{x^2 - a^2}{x - a}, & x \neq a \\ 8, & x = a \end{cases}$$

6. Use the graph to determine:



e) Discuss the continuity of the function. (give reasons for each point of discontinuity)

7. True or false: Give a brief explanation.

a. The graph of a rational function has at least one vertical asymptote.

b. If $\lim_{x \to c} f(x) = L$, then f(c) = L.

c. If a function is continuous at a point, then it is differentiable at that point.

8. Using the definition of derivative
$$\begin{pmatrix} f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \end{pmatrix}$$
, find the derivative of:
a. $f(x) = 4x^2$

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$$f(x) = \frac{1}{x-2}$$

9. The graph of f is given. Sketch the graph of the derivative on the same graph.



$$f(0) = 4; f'(0) = 0;$$

$$f'(x) < 0 \text{ for } x < 0$$

10. Identify a function f that has $f'(x) > 0 \text{ for } x > 0$. Sketch the function.

- 11. The figure shows the graph of g $^{\prime}$:
- a. g'(0) =
- b. g'(3) =
- c. What can you conclude about the graph of g knowing that $g'(1) = \frac{-8}{3}$?
- d. Is g(6) g(4) positive or negative? Explain.

e. Is it possible to find g(2) from the graph? Explain.



12. Find the derivative of the function (using the rules).

a.
$$y = 3x^2 - 5\sqrt[3]{x} + 6 + \frac{4}{x}$$

b. $f(x) = 5x^3 cosx$

c.
$$f(x) = \frac{2x + x^3 - x^5}{4x}$$
 d. $f(x) = \frac{3x^2}{x+4}$

e.
$$f(x) = xsecx - \frac{3}{x} + 2tanx$$
 f. $f(x) = sin^{2}x + cos^{2}x$

13. Find the equation of the line that is tangent to $f(x) = \frac{x}{x+4}$ at (-5, 5). Use derivative rules.

14. Determine the point(s) at which the graph of f(x) = x + sinx $0 \le x \le 2\pi$ has a horizontal tangent line.

15. The wind chill is the temperature, in degrees Fahrenheit, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour. If the air temperature is $32^{0}F$, then the wind chillis given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

a. Find W'(20). Using correct units, explain the meaning of W'(20) in terms of the wind chill.

b. Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.